

Spontaneous emission of many-body Schrödinger cats in metamaterials with large fine structure constant

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Understanding the quantum nature of light in situations where it interacts very strongly with matter is a question that is becoming experimentally relevant for qubits coupled to new classes of metamaterials with impedance close to the quantum limit. This problem enters the realm of many-body physics because the effective fine structure constant reaches values of order one, so that perturbative methods and concepts of quantum optics cannot be simply borrowed. Remarkably, describing the time-dependent state vector in terms of quantum-superposed classical (coherent) states provides not only a very efficient computational tool, but also a simple way to rationalize the physics at play. By analyzing the Wigner distribution of the radiated field, we find that spontaneous atomic decay does not lead to the standard emission of one-photon Fock states, but rather to the release of many-body Schrödinger cats.

Introduction. Quantum optics deals at a fundamental level with the properties of light emitted by single coherent sources, such as atoms and quantum dots. The smallness of the light-matter coupling provides a simple ground where photons constitute the basic granular elements of the electromagnetic radiation. Indeed, for atomic decay in three-dimensional space [1], the dimensionless coupling constant $\alpha = (\mathcal{P}/e\lambda)^2 \alpha_{\text{QED}}$ appears as a combination of two small factors, the ratio of the atomic dipole \mathcal{P} to the photon wavelength λ , and the universal value $\alpha_{\text{QED}} \simeq 1/137$ of the fine structure constant in vacuum. For simple atomic transitions, dimensional analysis gives the estimate $\mathcal{P}/e\lambda \lesssim \alpha_{\text{QED}}$, so that the light-matter coupling $\alpha \lesssim (\alpha_{\text{QED}})^3 \simeq 10^{-6}$ is vanishingly small.

The advent of microwave quantum optics in circuit-QED, where superconducting elements constitute a low-loss medium for the propagation of photons, has seen tremendous progresses in tunability [2]. In the case of one-dimensional waveguides, the coupling constant remains however of the order of $\alpha_{\text{QED}} \ll 1$, a regime that was explored only very recently [3–9]. Thus, reaching larger coupling in superconducting waveguides can only be attained by tweaking the fine structure constant itself. This requires to use superconducting metamaterials with high impedance, as can be intuitively understood by rewriting $\alpha_{\text{QED}} = Z_{\text{vac.}}/2R_K$ with $Z_{\text{vac.}} = 1/\epsilon_0 c \simeq 376 \Omega$ the vacuum impedance and $R_K = e^2/h \simeq 25812 \Omega$ the von Klitzing quantum of resistance. Metamaterials with characteristic impedance up to the order of R_K are presently explored following two strategies, using either long chains of Josephson junction [10–13] or disordered superconducting nanowires [14, 15]. Here, a huge kinetic inductance provides the necessary boost of the circuit impedance that allows to reach values of the light-matter interaction of order one. This new regime leads to anomalous scattering properties of photons [16–19] impinging on a non-trivial many-body vacuum [20]. This Letter aims at answering a seemingly simple question: is the one-photon state still well defined as quantum of radiation from a single emitter

in a high impedance medium?

We argue here that both methods and concepts of quantum optics have to be seriously revised to adapt to this new physical regime, which enters in fact the realm of many-body physics. This can be understood by a simple example: in standard quantum optics, coupled atomic levels are strongly dressed by states of light differing by exactly one quasi-resonant photonic mode [21]. However, as the coupling constant becomes of order one, the dressing effect [20, 22, 23] becomes strongly non-dispersive and involves a mean number of photons $\bar{n} \propto 1/(1 - \alpha)$ which becomes very large because the large fine structure constant domain is intrinsically broadband. In contrast to cavity-QED at ultra-strong coupling [24, 25], the Hilbert space becomes unmanageable for high impedance superconducting waveguides, because a large number of electromagnetic modes are involved, typically $N_{\text{modes}} = 1000$ for very long chains of Josephson junctions. The full quantum mechanical problem requires to tackle $(N_{\text{modes}})^{\bar{n}}$ quantum states, leading to a computational cost of order $(N_{\text{modes}})^{3\bar{n}}$, which can only be done for $\bar{n} \leq 1$, namely $\alpha \ll 1$ for long waveguides (this constitutes the standard Wigner-Weisskopf theory of quantum optics [1], based on a truncation to a maximum of a single Fock excitation). When already $\alpha \gtrsim 0.1$, only advanced many-body methodologies such as ML-MCTDH [26], TD-NRG [27, 28] or TD-DMRG [29, 30] can hope to address reliably the problem of many-body non-equilibrium unitary dynamics raised for standard quantum optics protocols (such as spontaneous emission or resonance fluorescence). These methodologies, which are still under development, show however some computational heaviness and offer limited physical intuition. To our knowledge, the radiation properties of a single atom emitting into such high impedance environment have not yet been fully elucidated.

In this Letter, we give a simple and unique answer to both the computational and physical questions raised here when extending quantum optics to an “universe” with large fine structure constant. We first show that

the natural concept of coherent states in a multimode electromagnetic field provides an incredibly efficient tool to compute and characterize the resulting many-body states of light. This does not come fully as a surprise, because such medium is highly dissipative, so that the selection of the most robust quantum states are the ones closest to the classical picture, the so-called pointer states [31]. More precisely, given a collection of N_{modes} electromagnetic modes with frequency ω_k and creation operator a_k^\dagger , a multimode coherent state is given by $|f\rangle = e^{\sum_{k=1}^{N_{\text{modes}}} [f^k a_k^\dagger - f^{k*} a_k]} |0\rangle$ in terms of a complex amplitude f^k corresponding to the mean expectation values of the electromagnetic field in mode k . Such unentangled state alone cannot describe the complexity of the many-body Hilbert space, but one can target in principle any quantum state, $|\Psi\rangle = \sum_{n=1}^{N_{\text{cs}}} p_n |f_n\rangle$, from a basic quantum superposition of a set of coherent states $\{f_n^k\}$ with respective weights p_n , for $n = 1 \dots N_{\text{cs}}$, with N_{cs} the number of coherent states allowed in the decomposition of the state vector $|\Psi\rangle$, which can thus be thought of as a multi-component multi-mode Schrödinger cat. One enlightening example of this decomposition consists in an arbitrary multimode single photon state that constitutes the basics of the Wigner-Weisskopf theory at $\alpha \ll 1$, and which can be re-expressed as $\sum_k C^k a_k^\dagger |0\rangle = [|f\rangle - |-f\rangle] / 2x$, with $f^k = x C^k$ in the limit $x \rightarrow 0$, using thus only two such multimode coherent states (this connection is mathematically obvious from a first-order Taylor expansion of the coherent state). Quite remarkably, previous work [20, 32–34] has demonstrated that the full many-body ground state of the ultra-strong matter-light Hamiltonian in waveguides can be described using this coherent state expansion, with only a polynomial scaling of the numerical resources as a function of the number of degrees of freedom N_{modes} . Our first goal here is to show that non-equilibrium unitary dynamics can be captured in a similar fashion, by providing a powerful algorithm that is able to simulate efficiently for the first time very large systems, up to thousands of electromagnetic modes. This unique technique allows us to consider and analyze the light spontaneously produced by a single emitter in a large impedance waveguide. We demonstrate that the radiated field can be characterized as a multimode Schrödinger cat, instead of the usual one-photon Fock state that is emitted in the quantum optics regime $\alpha \ll 1$. This striking observation reflects the unusual physics demonstrated when turning the fine structure constant to large values.

Methodology. We will specify our study to the standard model of waveguide-QED, described by the Hamiltonian:

$$H = \frac{\Delta}{2} \sigma_x + \sum_k \omega_k a_k^\dagger a_k - \sigma_z \sum_k \frac{g_k}{2} (a_k^\dagger + a_k), \quad (1)$$

defining the light-matter coupling g_k of a given mode to the two-level atom, which is described by Pauli matrices and a level splitting Δ . In contrast to standard quantum optics notation, we have intentionally written the atomic spectrum in the σ_x basis, in order to emphasize the natural

selection of coherent states caused by the σ_z coupling to the $|\uparrow\rangle$ and $|\downarrow\rangle$ states. Hence, the bare atomic ground state is $|g\rangle = [|\uparrow\rangle - |\downarrow\rangle] / \sqrt{2}$ in our notation.

In practice, the overall dimensionless coupling strength α can be encapsulated from the spectral density $J(\omega) = \pi \sum_k g_k^2 \delta(\omega - \omega_k) = 2\pi\alpha\omega e^{-\omega/\omega_c}$, with ω_c a high energy cutoff. For simplicity, we assume here a linear dispersion relation $\omega_k = k$ (the speed of light in the metamaterial is taken to unity in what follows). Following the previous discussion, we represent the state vector at an arbitrary time by a coherent state expansion:

$$|\Psi(t)\rangle = \sum_{n=1}^{N_{\text{cs}}} [p_n(t) |f_n(t)\rangle |\uparrow\rangle + q_n(t) |h_n(t)\rangle |\downarrow\rangle], \quad (2)$$

where the set of variables $v = \{p_n, q_n, f_n^k, h_n^k\}$ are complex and time dependent. The exact Schrödinger dynamics controlled by Hamiltonian (1) can be represented from a real Lagrangian density $\mathcal{L} = \langle \Psi(t) | \frac{i}{2} \vec{\partial}_t - \frac{i}{2} \overleftarrow{\partial}_t - \mathcal{H} | \Psi(t) \rangle$, when applying the variational principle $\delta \int dt \mathcal{L} = 0$ upon arbitrary variations of the state vector [35]. The simple representation (2) then results in Euler-Lagrange equations $\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial v} = \frac{\partial \mathcal{L}}{\partial v}$ for our set of variables, which read:

$$-i \frac{\partial E}{\partial p_j^*} = \frac{1}{2} \sum_m (2\dot{p}_m - p_m \kappa_{mj}) \langle f_j | f_m \rangle, \quad (3)$$

$$\begin{aligned} -i \frac{\partial E}{\partial f_j^{k*}} = & -\frac{1}{4} \sum_m (2\dot{p}_m - p_m \kappa_{mj}) p_j^* (f_j^k - 2f_m^k) \langle f_j | f_m \rangle \\ & + \frac{1}{4} \sum_m (2\dot{p}_m^* - p_m^* \kappa_{mj}^*) p_j f_j^k \langle f_m | f_j \rangle \\ & + \sum_m p_m p_j^* \dot{f}_m^k \langle f_j | f_m \rangle. \end{aligned} \quad (4)$$

Identical equations are obtained for the variables q_n and h_n^k . We have denoted here $E = \langle \Psi | H | \Psi \rangle$ the average energy, which is conserved throughout the dynamics, and introduced an important parameter in what follows:

$$\kappa_{mj} = \sum_{k' > 0} [\dot{f}_m^{k'} f_m^{k'*} + \dot{f}_m^{k'*} f_m^{k'} - 2f_j^{k'*} \dot{f}_m^{k'}]. \quad (5)$$

In contrast to the dynamics with a single coherent state [36, 37], one encounters here a computational difficulty [38], because the time-derivatives $\dot{f}_m^{k'}$ of all possible coherent state amplitudes enter the dynamical equation ruling a given field f_j^k in Eq. (4), through the parameter κ_{mj} in Eq. (5). Indeed, for stability reasons it is crucial to formulate the dynamical equations in an explicit form $\dot{f}_j^k = F[v]$, where F is only a functional of the variables $v = \{p_n, q_n, f_n^k, h_n^k\}$ without reference to their time derivatives. Inverting the system of equations numerically in order to bring it into explicit form is however prohibitive (unless the number of modes is small, for instance in the case of the Wilson discretization [39], which is however not adapted to study properly the dynamics within the bath), since the inversion alone would cost

$(N_{\text{modes}} \times N_{\text{cs}})^3$ operations. Luckily, a very efficient trick allows to make the inversion in $(N_{\text{cs}})^6$ operations, which is favorable provided $N_{\text{cs}} \ll N_{\text{modes}}$, as is the case for very long Josephson arrays or broadband environments. First, we invert Eq. (3-4) by making explicit the dependence in κ_{nj} :

$$\dot{p}_i = \sum_{jn} A_{ijn} \kappa_{nj} + G_i, \quad (6)$$

$$\dot{f}_i^s = \sum_{jn} B_{ijns} \kappa_{nj} + H_{is}, \quad (7)$$

defining the compact notation $A_{ijn} = \frac{1}{2} M_{ij}^{-1} p_n \langle f_j | f_n \rangle$, $G_i = \sum_j (M_{ij}^{-1} P_j)$, $B_{ijns} = \frac{1}{2} (N^{-1})_{ij} p_n f_n^s \langle f_j | f_n \rangle - \sum_{lm} (N^{-1})_{il} A_{mjn} f_m^s \langle f_l | f_m \rangle$, and $H_{is} = \sum_j (N^{-1})_{ij} F_j^s - \sum_{jm} (N^{-1})_{ij} G_m f_m^s \langle f_j | f_m \rangle$, with $P_j = -i \partial E / \partial p_j^*$, $F_j^k = -i \partial E / \partial f_j^{k*} + (1/2) [P_j p_j^* + P_j^* p_j] f_j^k$, and the overlap matrices $M_{jm} = \langle f_j | f_m \rangle$ and $N_{jm} = p_m \langle f_j | f_m \rangle$. The evolution equations (6-7) are still not in explicit form, because the parameters κ_{nj} explicitly depend on time derivatives in Eq. (5). However, we can now use Eq. (7) to replace all the \dot{f}_i^k terms in Eq. (5), which gives a closed equation for the κ matrix:

$$\begin{aligned} \kappa_{im} = & \sum_{jns} \left[(f_i^{s*} - 2f_m^{s*}) B_{ijns} \kappa_{nj} + f_i^s B_{ijn s}^* \kappa_{nj}^* \right] \\ & + \sum_s \left[f_i^{s*} H_{is} + f_i^s H_{is}^* - 2f_m^{s*} H_{is} \right]. \end{aligned} \quad (8)$$

Inverting this linear systems with $(N_{\text{cs}})^2$ parameters provides the final $(N_{\text{cs}})^6$ scaling of our algorithm. Of course the gain is important only provided that N_{cs} stays small during the unitary time evolution. We show now that the dynamics indeed converges rapidly for a surprisingly small number of coherent states, demonstrating that the decomposition (2) is physically well motivated.

Atomic decay. In what follows, we consider the issue of spontaneous atomic decay into a very long high impedance medium, a problem [19] that has not yet revealed all its fascinating physics, as we will demonstrate. The dynamics thus starts with the initial wavefunction $|\Psi(t=0)\rangle = |0\rangle \otimes [|\uparrow\rangle + |\downarrow\rangle] / \sqrt{2}$, where the environment is set in its bare vacuum, and the two-level system is taken in the excited atomic configuration (we recall that the two-level atom is described by a σ_x coupling in our notation, see Eq. (1)). The system evolves unitarily at later times according to Hamiltonian (1). For $\alpha \ll 1$, one expects spontaneous emission of a single photon and decay of the atom towards its ground state. The converged result for $\alpha = 0.1$, seen in the upper panel of Figure 1, shows indeed the expected decay behavior. Here the atom decays from the initial value $\langle \sigma_x(0) \rangle = 1$ to a final stationary state with $\langle \sigma_x(\infty) \rangle \simeq -0.9$. In contrast to the quantum optics regime with $\alpha \ll 1$, the long-time value of the atomic occupation turns out to be different from -1, because the atom assumes a dressing by the photons in its ground state for finite α values, an effect that is not captured

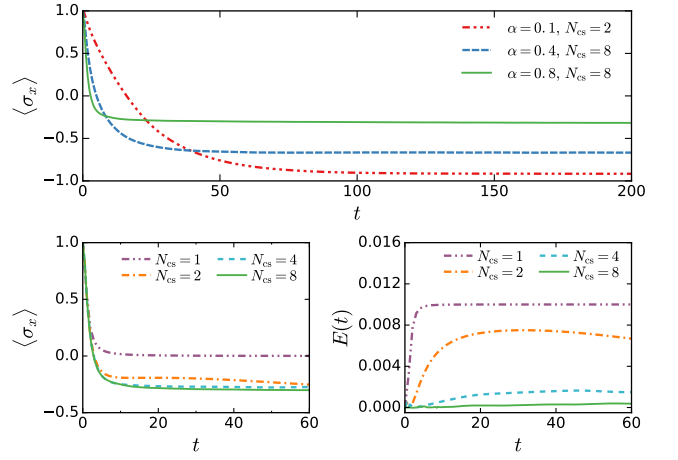


FIG. 1. (Color online) Top panel: Atomic decay $\langle \sigma_x(t) \rangle$ as a function of time, for increasing dimensionless impedance $\alpha = 0.1, 0.4, 0.8$ and a respective number of coherent states $N_{\text{cs}} = 2, 8, 8$ that is sufficient for good convergence (in all computations, $\Delta/\omega_c = 0.2$ and $N_{\text{modes}} = 1600$). Bottom left panel: The rapid convergence of $\langle \sigma_x(t) \rangle$ is demonstrated for the challenging case $\alpha = 0.8$, with an increasing number of coherent states $N_{\text{cs}} = 1, 2, 4, 8$ in the decomposition (2). Bottom right panel: error $E(t)$ with respect to the exact Schrödinger dynamics, which drops rapidly to zero with increasing N_{cs} .

by the standard RWA. This renormalization effect of the ground state occupation is even more pronounced at larger dissipation strength [16, 32], as seen by the curves for $\alpha = 0.4, 0.8$ in the upper panel of Fig. 1.

In order to prove convergence of our results, we provide in the lower left panel of Fig. 1 the evolution of the atomic decay for an increasing number of coherent states in the Ansatz (2), and in the lower right panel a measure of the error $E(t)$ with respect to the exact Schrödinger dynamics. For this purpose, we define a new vector $|\Phi(t)\rangle \equiv (i\partial_t - H)|\Psi(t)\rangle$ from the state vector $|\Psi(t)\rangle$, which ought to be strictly zero if the dynamics was perfectly captured. The error is thus defined by the norm $E(t) \equiv \langle \Phi(t) | \Phi(t) \rangle$, and rapidly drops to zero by increasing to moderate values the number of coherent states N_{cs} . Thus, our coherent state method provides numerically exact results in all regimes of coupling for small computational effort.

Nature of the emitted spontaneous radiation. Having benchmarked our methodology, we can now address the physical properties of the electromagnetic waves spontaneously emitted by the atom during a radiative process, since we have at our disposal, from the decomposition (2), the complete wavefunction of the joint atom and waveguide system. For this purpose, we plot in real space on Fig. 2 the real part of the coherent state amplitudes $f_1(x, T)$, $f_2(x, T)$, $h_1(x, T)$, and $h_2(x, T)$ associated to the largest weights p_1 and p_2 in Eq. (2). Here we choose a time $T \gg L_K$ long enough that the system has fully relaxed to its many-body ground state, where $L_K = (\omega_c/\Delta)^{\alpha/(1-\alpha)}$ is the spatial extent of the entanglement cloud [20]. Due to the obvious \mathbb{Z}_2 symmetry (transforming $a^\dagger \rightarrow -a^\dagger$ and

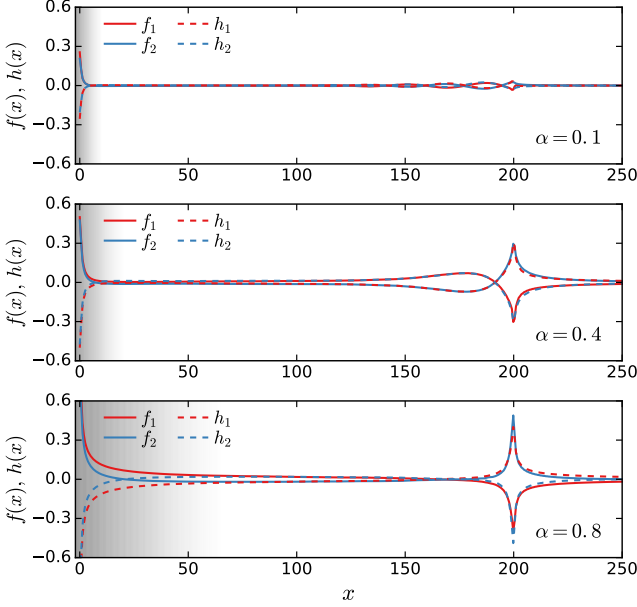


FIG. 2. (Color online) Real part of the coherent state amplitudes in real space $\text{Re}[f_n(x, T)]$ and $\text{Re}[h_n(x, T)]$ (for $n = 1, 2$) at a given time $T = 200$ long enough that the emitted wavepacket is disentangled from the short distance entanglement cloud (denoted in shaded area). Here $\Delta/\omega_c = 0.2$, and $N_{\text{cs}} = 2$, with $\alpha = 0.1$ (top panel), $\alpha = 0.4$ (middle panel), and $\alpha = 0.8$ (bottom panel).

$\sigma_z \rightarrow -\sigma_z$) of Hamiltonian (1) that is also preserved by our initial wavefunction $|\Psi(0)\rangle$, we have the exact symmetry relations $f_n(x, t) = -h_n(x, t)$ and $p_n(t) = q_n(t)$ for all n at all times. In order to ease the physical interpretation of these curves, let us first discuss the short distance behavior of the coherent state amplitudes. For $x < L_K$, one sees a static entanglement cloud between the atom and its surrounding modes in the waveguide. This cloud penetrates clearly deeper and deeper within the waveguide as can be seen by comparing the upper and lower panels of Fig. 2 obtained for $\alpha = 0.1$ and $\alpha = 0.4$ respectively. Within this short distance cloud, the coherent state amplitudes can be seen to obey (in addition to the exact \mathbb{Z}_2 symmetry mentioned above) the approximate relations: $f_1(x, t) \simeq f_2(x, t) \equiv f_{\text{cloud}}(x)$ and $h_1(x, t) \simeq h_2(x, t) \equiv -f_{\text{cloud}}(x)$ for $x < L_K$, and $p_1 = -p_2 \equiv p$ (not shown). In contrast, within the emitted wavepacket at large distance, a different set of relations is seen, namely $f_1(x, t) \simeq h_2(x, t) \equiv f_{\text{wp}}(x - t)$ and $h_1(x, t) \simeq f_2(x, t) = -f_{\text{wp}}(x - t)$ for $x \gg L_K$. Note here that the wavepacket is traveling away from the impurity without changing its shape (once outside the Kondo cloud) due to the linear dispersion relation, hence the trivial temporal dependence in these expressions. Now, by factoring the coherent state in real space $|f_1\rangle = |f_{\text{cloud}}\rangle \otimes |f_{\text{wp}}\rangle$, one readily finds that the long-time complete wavefunction (2)

is well approximated as follows:

$$\begin{aligned} |\Psi(\infty)\rangle &\simeq [p_1|f_1\rangle + p_2|f_2\rangle]|\uparrow\rangle + [q_1|h_1\rangle + q_2|h_2\rangle]|\downarrow\rangle \\ &\simeq [p|f_{\text{cl}}\rangle|f_{\text{wp}}\rangle - p|f_{\text{cl}}\rangle|-f_{\text{wp}}\rangle]|\uparrow\rangle + \\ &\quad [p|-f_{\text{cl}}\rangle|-f_{\text{wp}}\rangle - p|-f_{\text{cl}}\rangle|f_{\text{wp}}\rangle]|\downarrow\rangle \\ &\simeq p[|f_{\text{cl}}\rangle|\uparrow\rangle - |-f_{\text{cl}}\rangle|\downarrow\rangle] \otimes [|f_{\text{wp}}\rangle - |-f_{\text{wp}}\rangle]. \end{aligned} \quad (9)$$

The last line in (10) can be straightforwardly interpreted as a factorization between the short distance cloud $|\Psi_{\text{cl}}\rangle \equiv |f_{\text{cl}}\rangle|\uparrow\rangle - |-f_{\text{cl}}\rangle|\downarrow\rangle$ (which comprises both the atom and its neighboring sites in the waveguide) and the emitted wavepacket $|\Psi_{\text{wp}}\rangle \equiv |f_{\text{wp}}\rangle - |-f_{\text{wp}}\rangle$ at large distances. This is physically expected as the system relaxes at long times to its many-body static ground state, while emitting a stream of electromagnetic radiation carrying the excess energy but no quantum correlations with the bound states. We stress that the above expressions for $|\Psi_{\text{cl}}\rangle$ and $|\Psi_{\text{wp}}\rangle$ are only approximate, as small quantum corrections arise at increasingly large α values [32, 33], which are fully accounted for by the terms $n > 2$ in the expansion (2).

We are now fully equipped to discuss the nature of the emitted radiation, based on the approximate expression $|\Psi_{\text{wp}}\rangle = |f_{\text{wp}}\rangle - |-f_{\text{wp}}\rangle$ for the emitted wavepacket, which is quite accurate provided α is not too large. Let us first consider a regime close to quantum optics with $\alpha = 0.1 \ll 1$. Here, one sees from the upper panel of Fig. 2 that $f_{\text{wp}} \ll 1$, so that the coherent states can be expanded at linear order, and the emitted wavepacket is well represented by a single-photon state $|\Psi_{\text{wp}}\rangle = 2 \sum_x f_{\text{wp}}(x) a^\dagger(x) |0\rangle$, in agreement with Wigner-Weisskopf theory. This simple interpretation can be confirmed by performing a Wigner tomography of the emitted wavepacket. In contrast to photons emitted into a cavity [21, 40], we stress here that the radiation is not purely monochromatic, due to the strong damping effect on the atom provided by the waveguide. As is standard practice in information treatment [41], one uses a temporal weighting of the output signal, which, owing to linear dispersion in the waveguide, can be defined by a spatial average, using an effective creation operator $b^\dagger = \sum_x w(x) a^\dagger(x) \Theta(x - L_K)$. Here the Θ -function allows to filter out the static bound component of the screening cloud. We find in practice that an efficient filtering window is given by monitoring the typical amplitude of the wavepacket, that we extract according to $w(x) = \langle \Psi | a^\dagger(x) (1/2 + \sigma_z/2) | \Psi \rangle$. Now, the Wigner distribution [21] can be computed following the standard expression $W(\lambda) = \int (d^2\beta/\pi^2) C_s(\beta) e^{\lambda\beta^* - \lambda^*\beta}$, with the symmetrized correlation function $C_s(\beta) = \langle \Psi | e^{\beta b^\dagger - \beta^* b} | \Psi \rangle$. For the weak dissipation regime $\alpha = 0.1 \ll 1$, the resulting phase space distribution is shown as the first panel of Fig. 3. As expected, the Wigner distribution possesses the characteristic behavior of a single Fock state, with a large region of negative “probability” near the origin and perfect isotropy in phase space. Intermediate dissipation with $\alpha = 0.4$, as shown in the middle panel of Fig. 2, leads to a sizeable coherent state amplitude $f_{\text{wp}}^{\text{max}} \simeq 0.3$. In this case, the linearization of the coherent state is no more le-

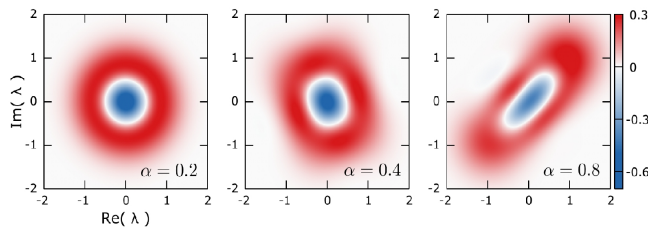


FIG. 3. (Color online) Wigner distribution $W(\lambda)$ as a function of $\text{Re}(\lambda)$ and $\text{Im}(\lambda)$ at increasing values of dissipation $\alpha = 0.1, 0.4, 0.8$ (left to right panel), which crosses over from a one-photon state to a multimode cat state.

gitimate, but an expansion to third order is still accurate, which turns out to be equivalent to a strong squeezing effect of the one-photon Fock state. This interpretation is confirmed by the shape of the Wigner distribution, seen in the second panel of Fig. 3, which shows an elongated one-photon state. Finally, the regime of large dissipation for $\alpha = 0.8$ enters deep in the Kondo domain [16, 17, 20],

with coherent state amplitudes becoming of order one, so that an interpretation of the radiation in terms of squeezing is not possible anymore. The Wigner distribution seen in the third panel of Fig. 3 demonstrates indeed the characteristic pattern of an odd Schrödinger cat state. In contrast to quantum optics protocols [21, 42], we stress that cat states are spontaneously radiated by a single emitter in a high impedance medium, a remarkable observation that constitutes the main findings of this Letter. In addition, these Schrödinger cats are truly many-body in nature, and involve a sizeable number of physically distinct degrees of freedom, typically dozens of neighboring Josephson elements in the superconducting circuit.

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